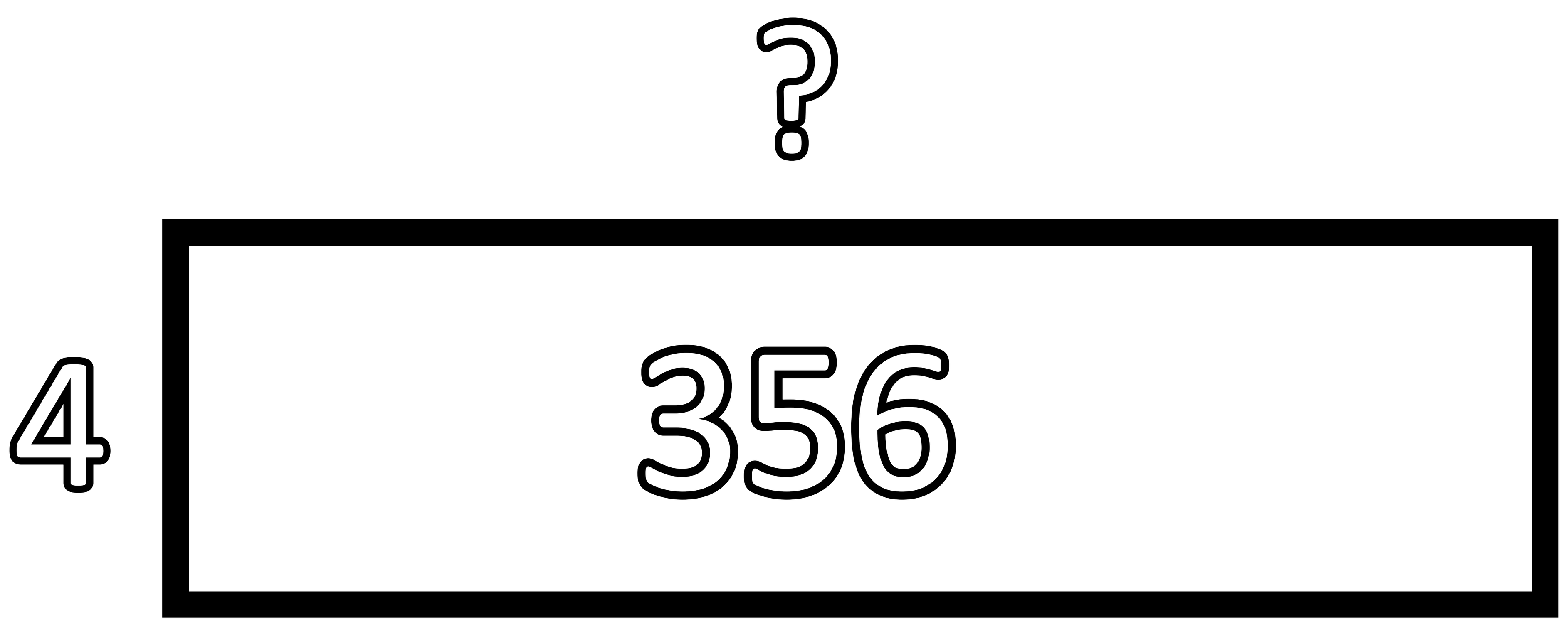
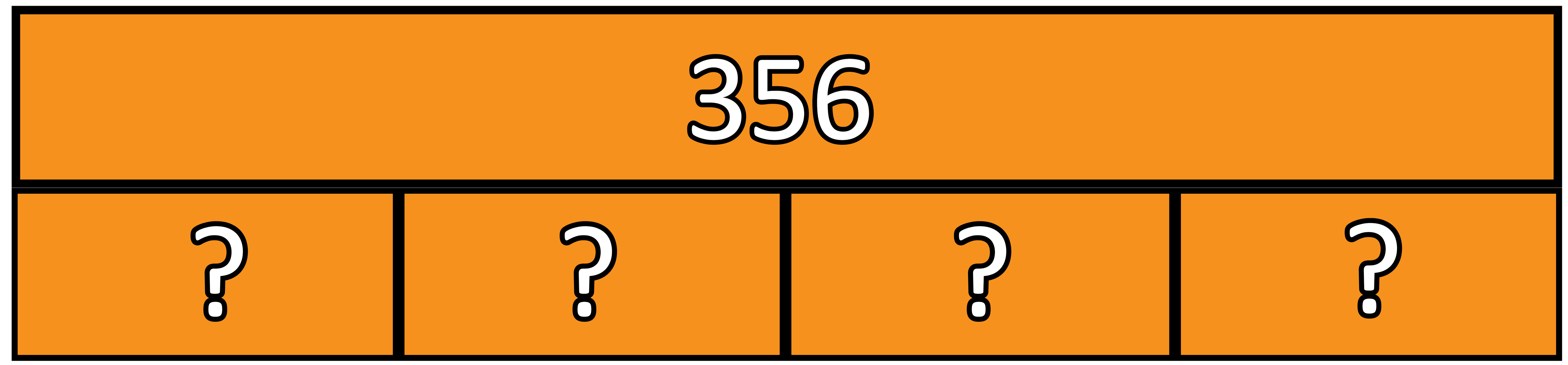
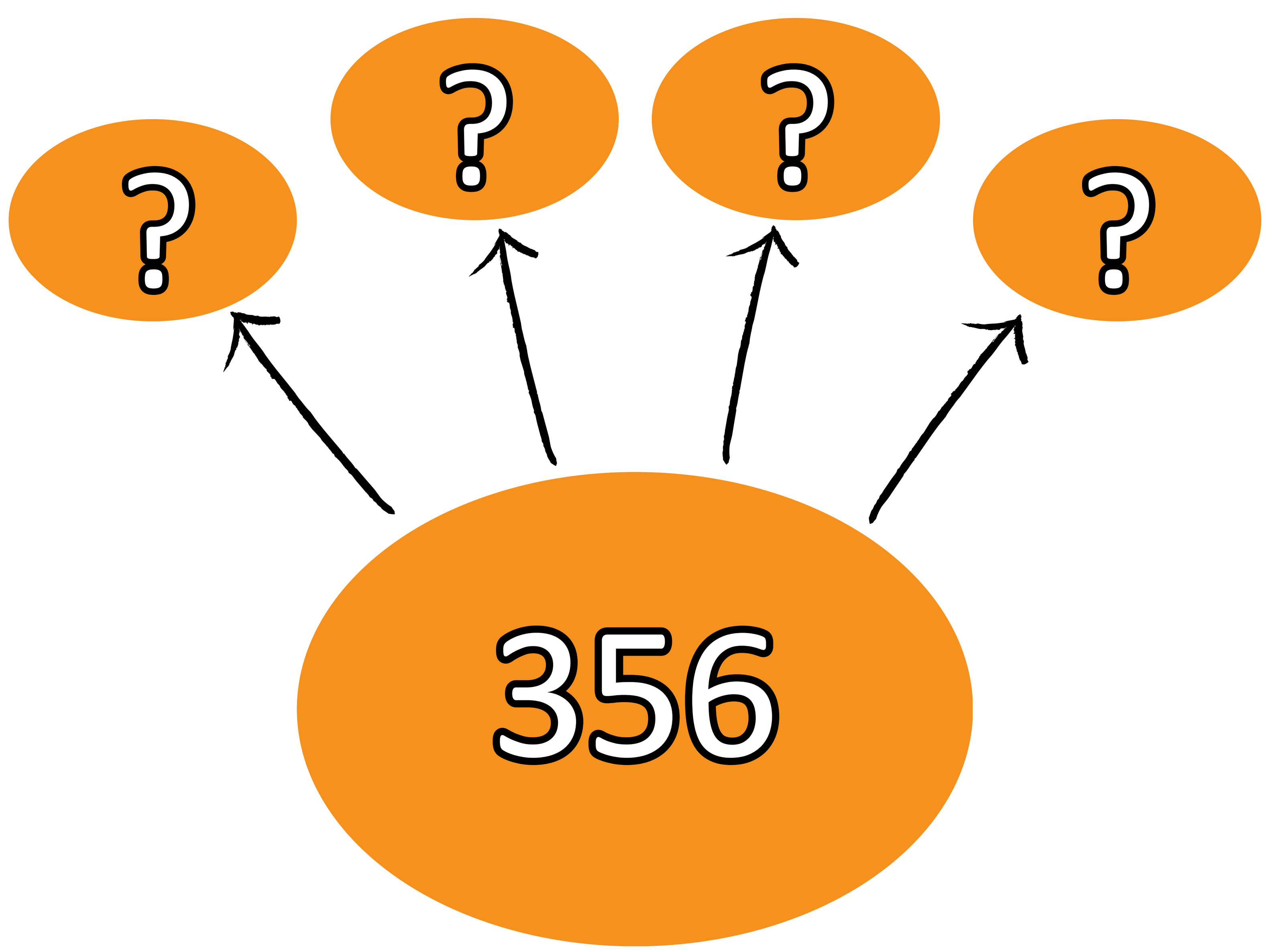
Visualizing Division $356 \div 4 = ?$

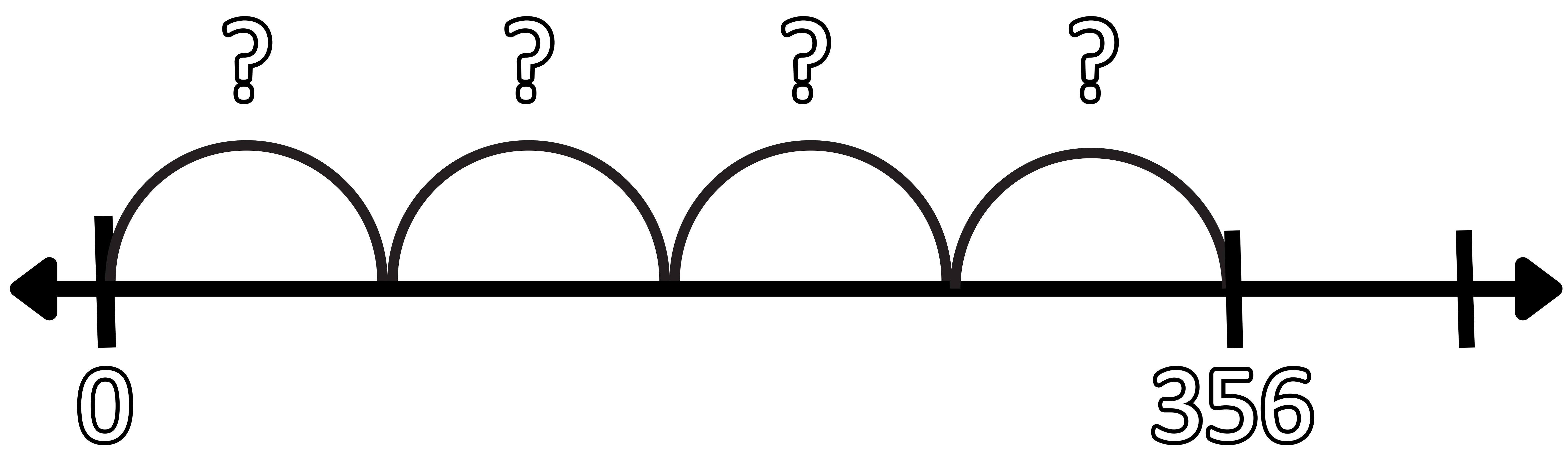


Open Area Model



Bar Model



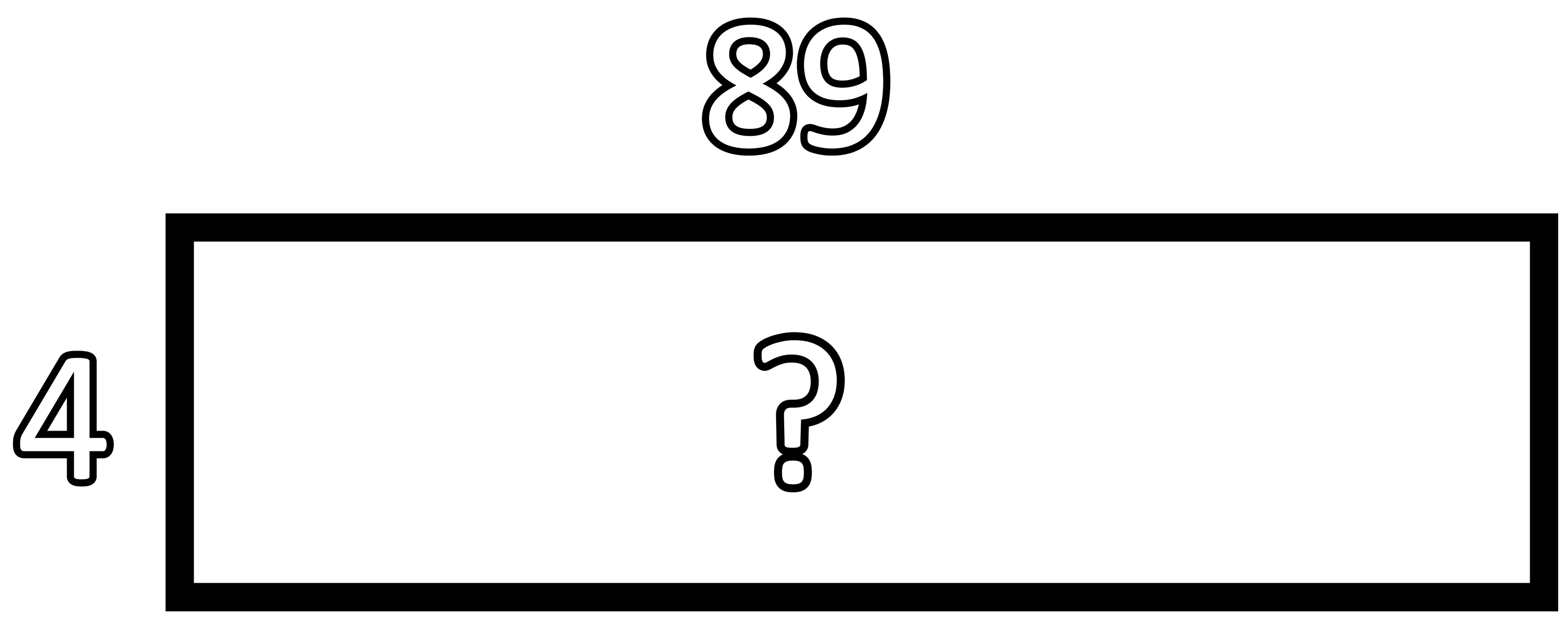


Open Number Line

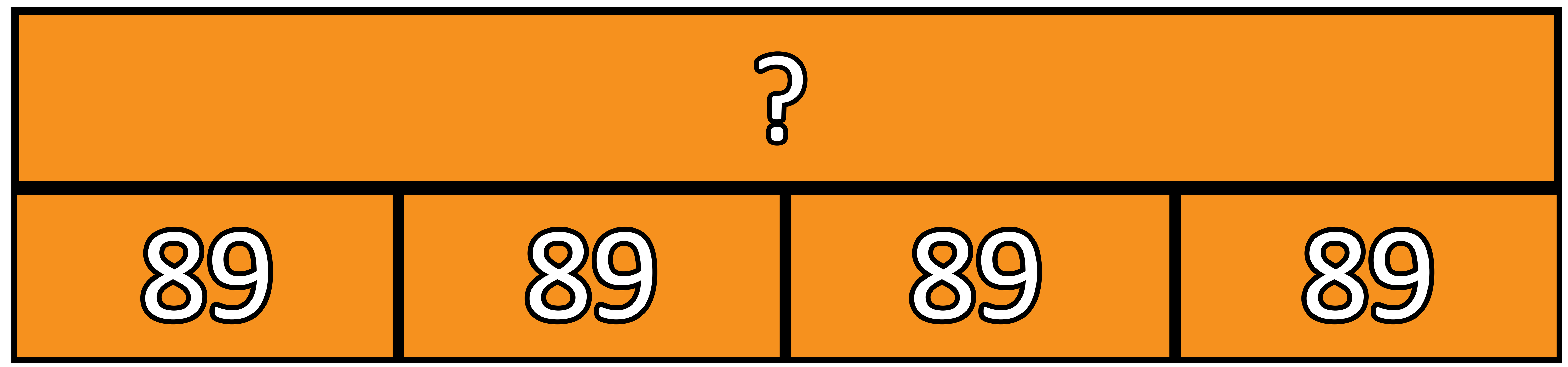
SD33 Numeracy

Visualizing Multiplication

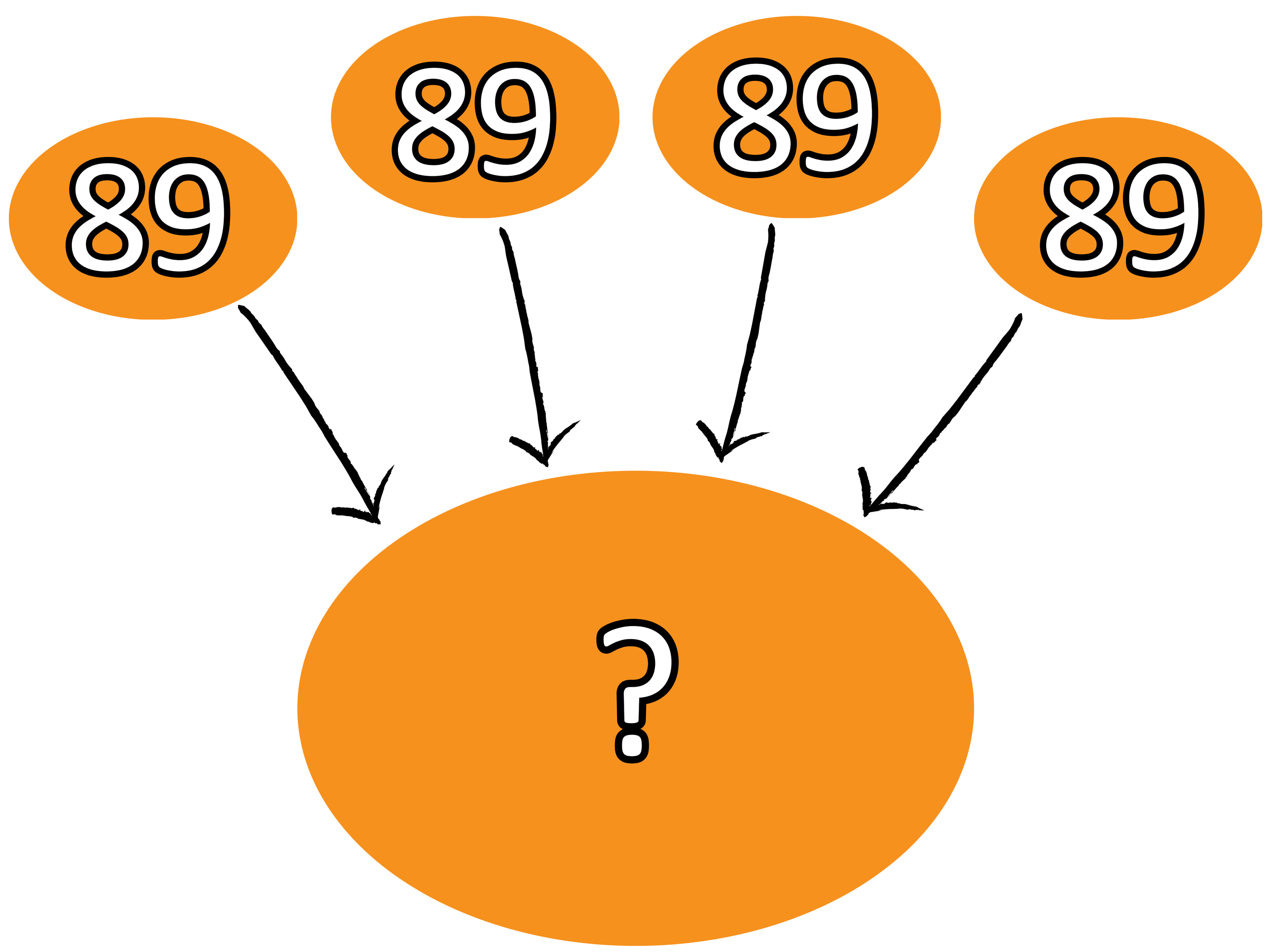
4x89=?

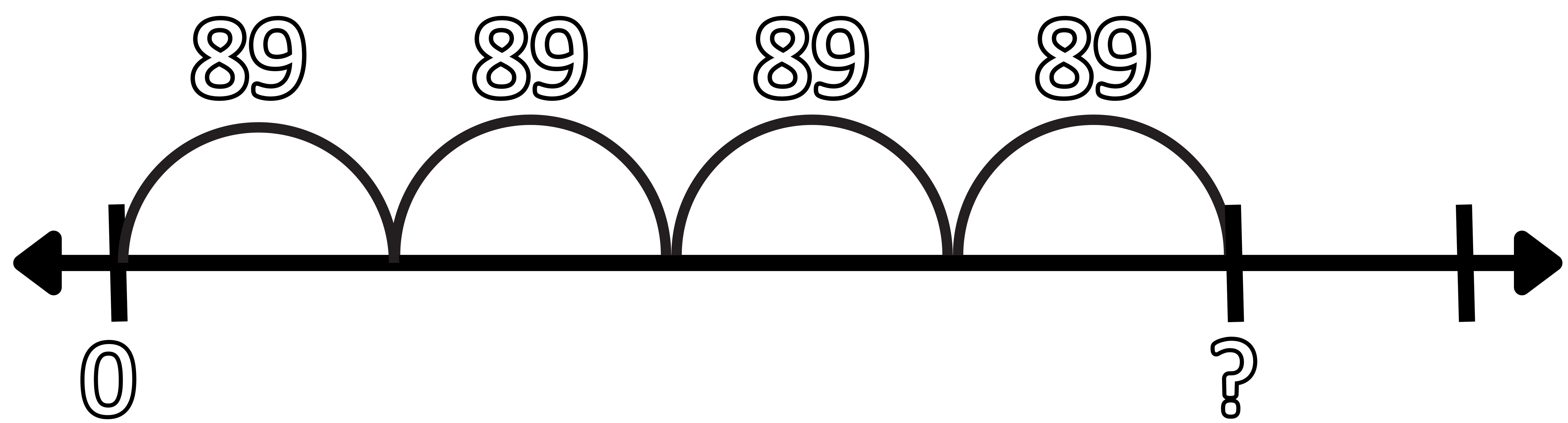


Open Area Model



Bar Model





Open Number Line

SD33 Numeracy

Four friends friends who owned an Xbox Series S together sold it for $356. If they split the money evenly, how much would each of them get?

$89

Which example shows division?

$89

Which example shows multiplication?

$356

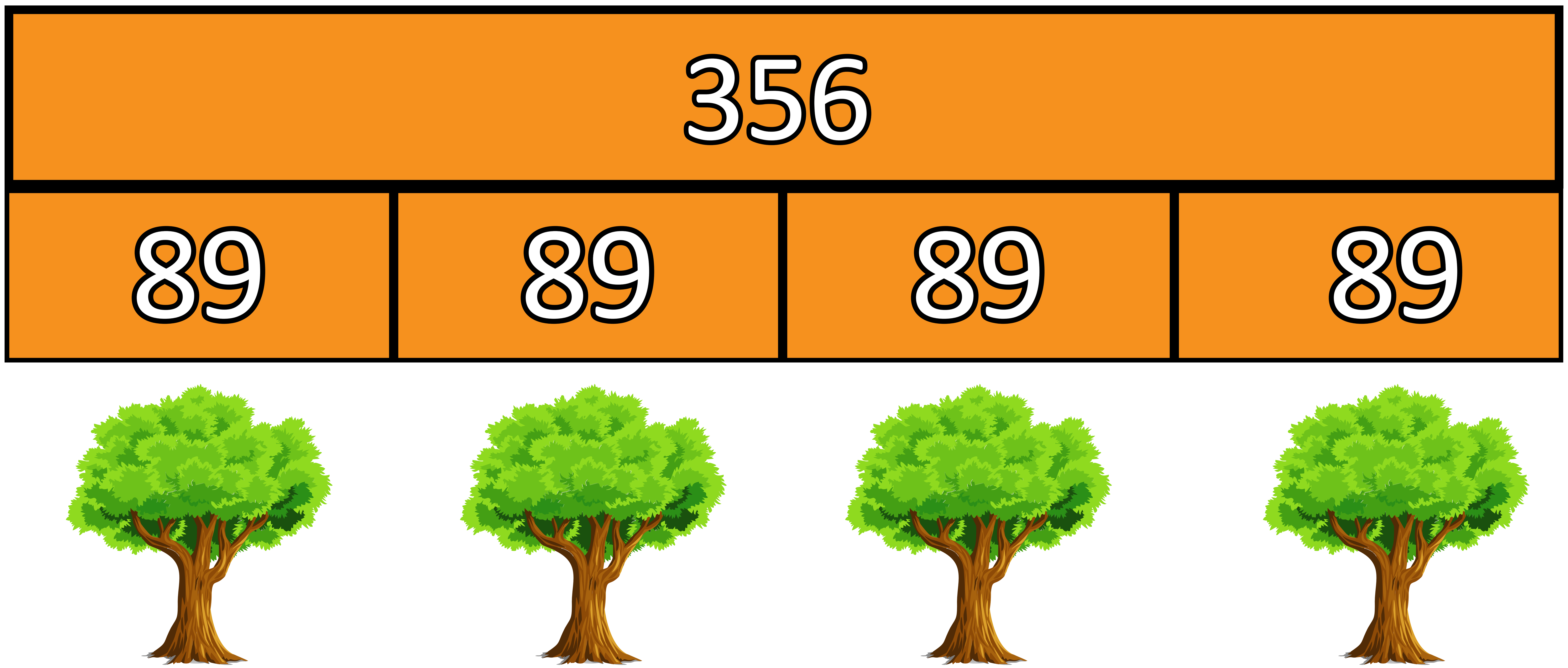
$89

Four friends each contributed $89 to buy an Xbox Series S. How much money do they have altogether?

$89

There were 4 trees and each one had 89 apples on it. How many apples were there altogether?

We picked 356 apples altogether. If each of the 4 trees grew the same number of apples, how many apples did we get from each one?



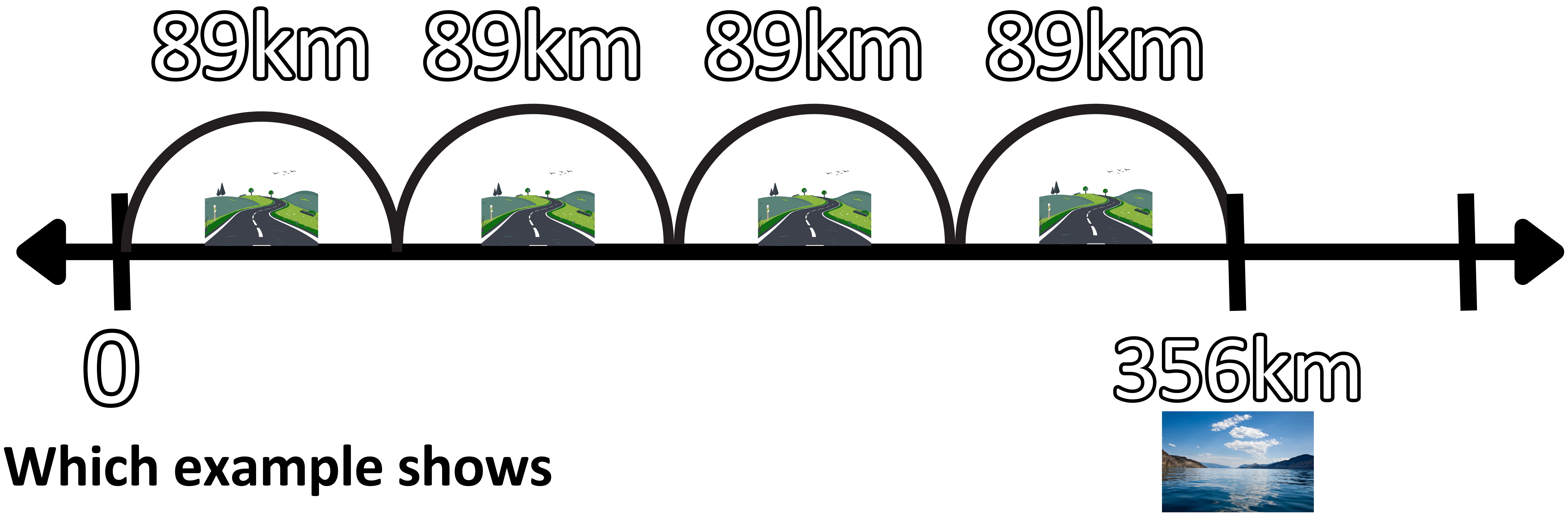
Which example shows division?

Which examples shows multiplication?

Can you see the connection to addition and subtraction?

A family wanted to split their drive to Penticton into 4 equal sections so they could have lots of breaks. If the total distance was 356km, how long would each section be?

A family wanted to split their drive to Penticton into 4 equal sections, with each one being 89km. How far will they drive in total?



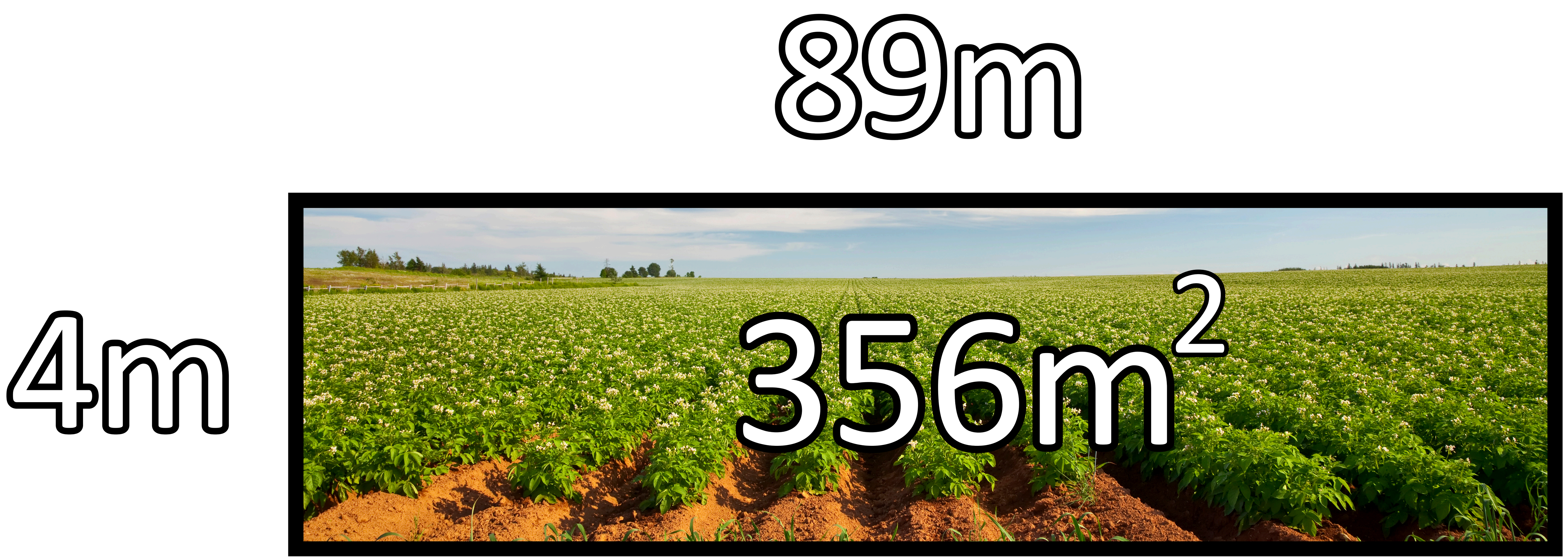
Which example shows division?

Which example shows multiplication?

Can you see the connection to addition and subtraction?

A farmer has 356 seed potatoes to plant and each one needs 1 square metre of space to grow. If she plants 4 rows, how many seed potatoes will she plant in each row?

If a farmer has a field that is 4m wide and 89m long, how many seed potatoes does she need to plant if each one needs 1 square metre of space?



Which example shows division?

Which example shows multiplication?

Partial Quotient Method for Division

Traditional Algorithm "Long Division"

Pros:

- Quickly produces an answer for students with all the skills to carry out the process

Cons:

- Requires students to know their multiplication facts to 10x10 (which is not required by the BC Curriculum until Grade 7)
- Requires students to follow a multistep process in the correct order with no mistakes
- Reasons that we divide, multiply, and subtract as part of the process are not clear to many students
- Does not clearly teach the conceptual understanding of division for most students
- Because the process is disconnected from the conceptual understanding, students often struggle to identify and correct errors
- If students lack any of the prerequisite skills to carry out the algorithm, then student has no path toward a correct solution and no chance of experiencing success
- Feelings of hopelessness in Math lead to disengagement and disruptive behaviour

Partial Quotient Method

Pros:

- The order of steps and the amount divided in each step is flexible and determined by the student, according to their readiness, confidence, and familiarity with the process
- Students with emergent fluency with multiplication facts can still be successful using more accessible numbers like 2, 5, 10, 20, and 50
- The structure of the algorithm pairs nicely with concrete representations (manipulatives) and pictorial representations (models/sketches) to teach conceptual understanding of division
- Becomes more efficient as students become more confident and skillful with the process (See next the next page for an example)

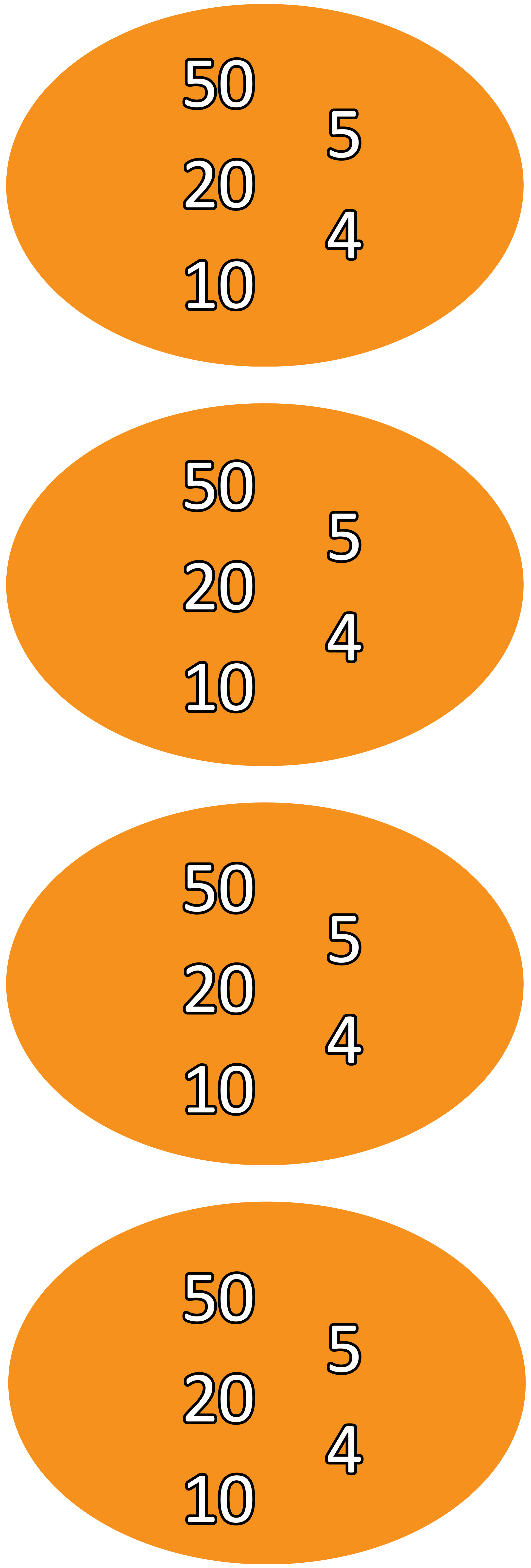
Cons:

- It's different than the way we learned in school

Go to the next page to see the flexibility of the Partial Quotient Method

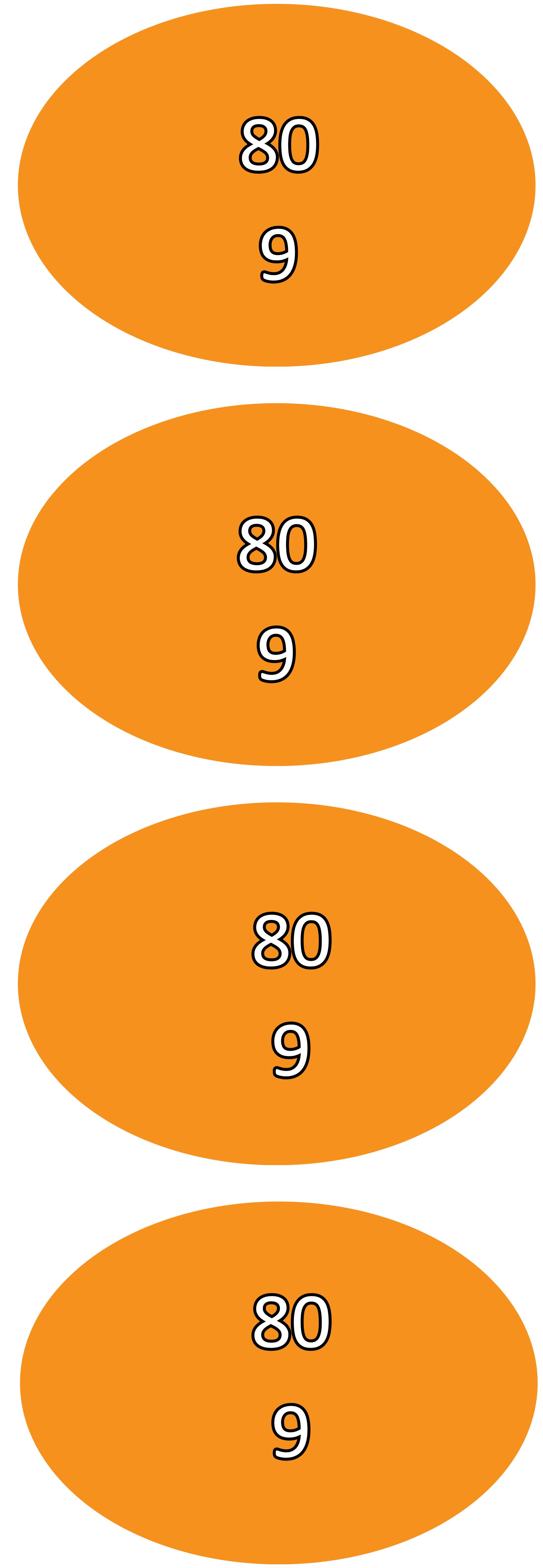
Example A - More accessible

Left	Each
4)356	
-200	50
156	
-80	20
76	
-40	10
36	
-20	5
16	
-16	+ 4
0	89



Example B - More efficient

Left	Each
4)356	
-320	80
36	
-36	+ 9
0	89



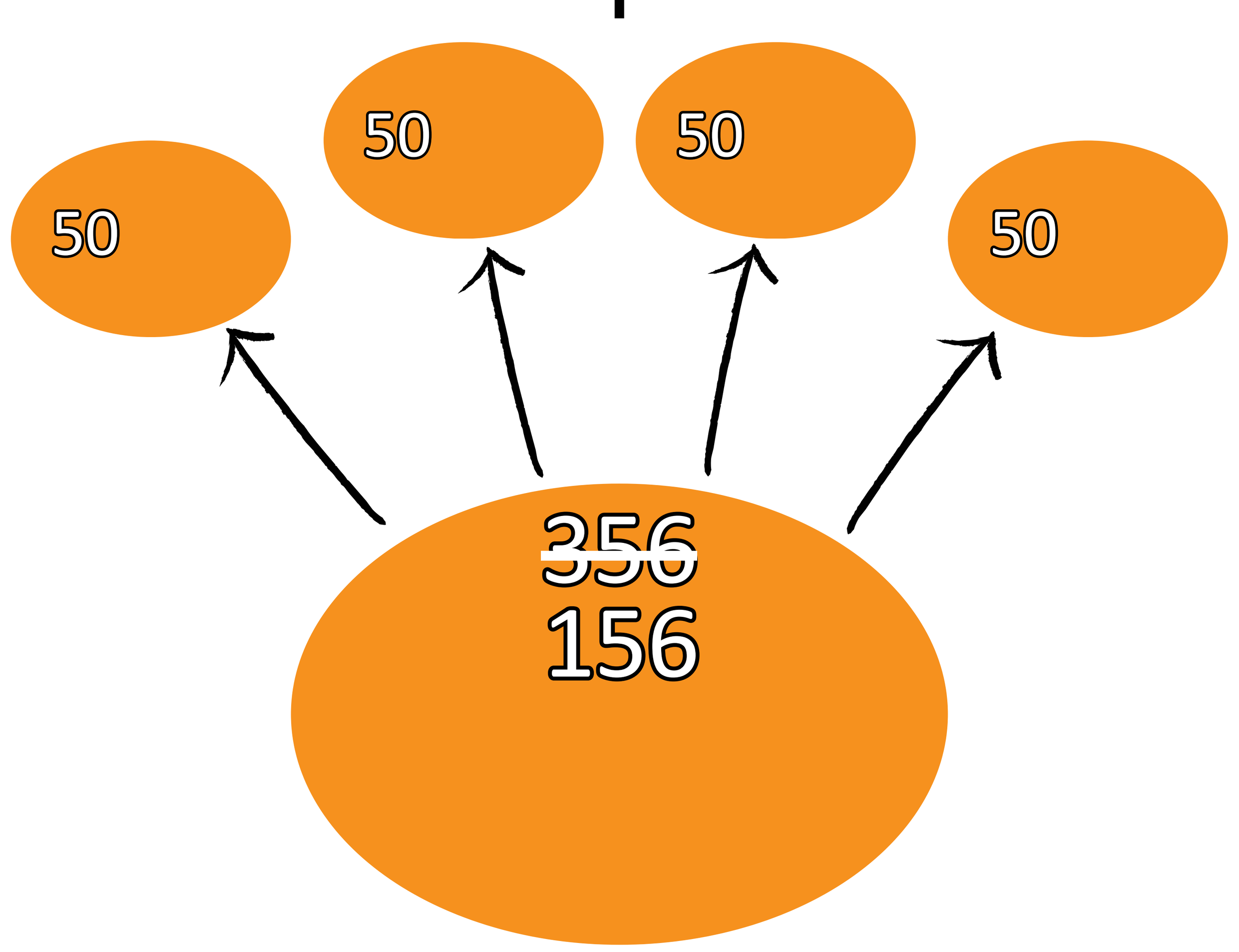
- Many students benefit from drawing a picture like the ones included here because it reminds them why they are doing each step (e.g. "I put 20 in each of the 4 groups, so that means I have to subtract 4x20=80 from my remaining total"). Most students will let go of the drawing as they get more comfortable using the algorithm.
- Example A shows the work of a more emergent mathematician. They aren't yet able to see that putting 80 into each group will get rid of most of their dividend (the big number that's being divided into groups). They are also only comfortable multiplying or skip counting by more accessible numbers like 5, 20, and 50. It took a while, but they were successful in coming to correct solution.
- Example B shows a more confident mathematician. They were able to use their number sense and computational fluency to find a correct solution even more efficiently than by using the traditional algorithm. Their success stemmed from their accumulated skills and understanding in math, rather than by their ability to reproduce a memorized process using memorized facts.
- In either example, students could have used different amounts in a different order and found the same solution in the end.

Steps in the Partial Quotient Method

Pictorial and Symbolic Representations

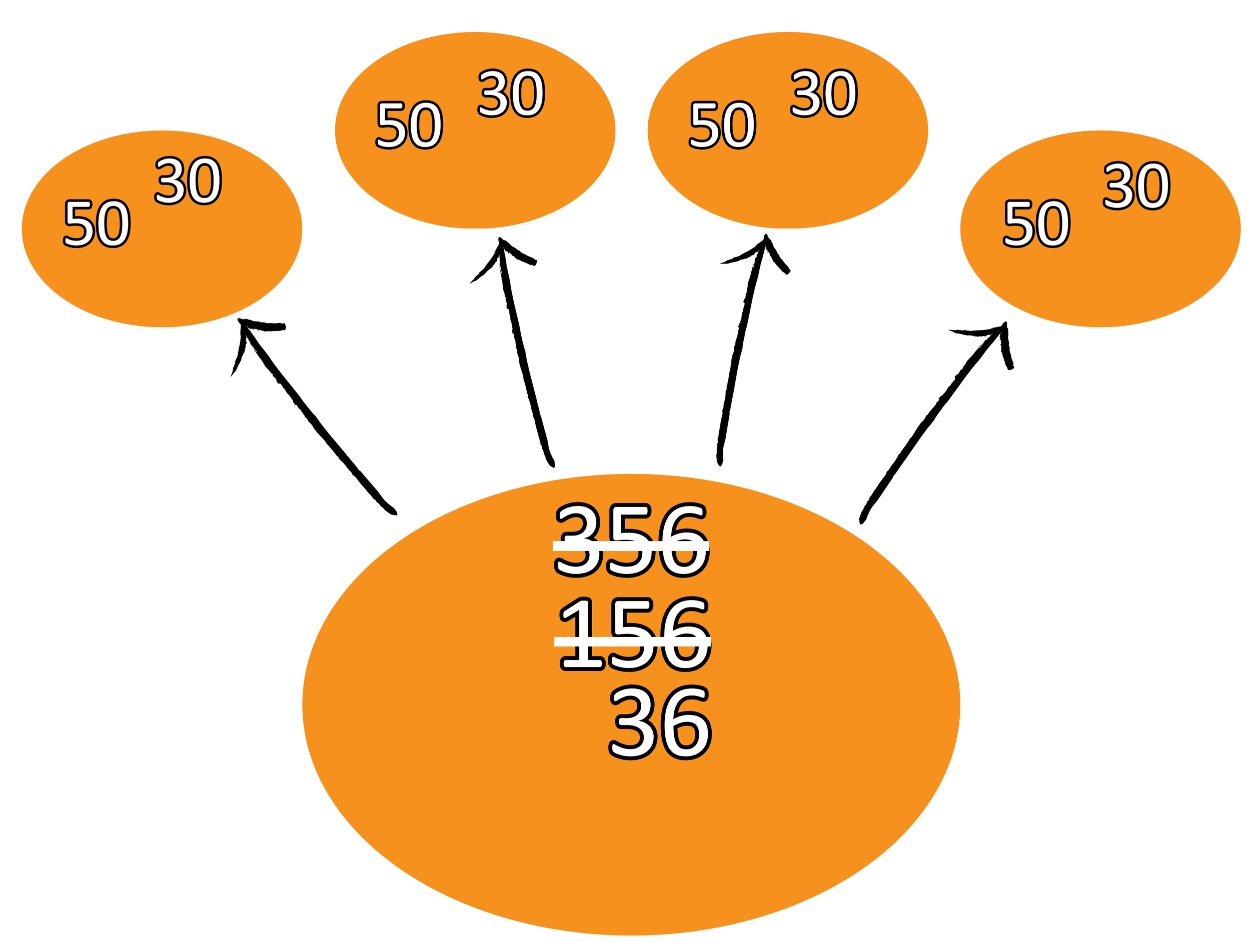
Step 1

	Left	Each
4)	356	
	-200	50
	156	



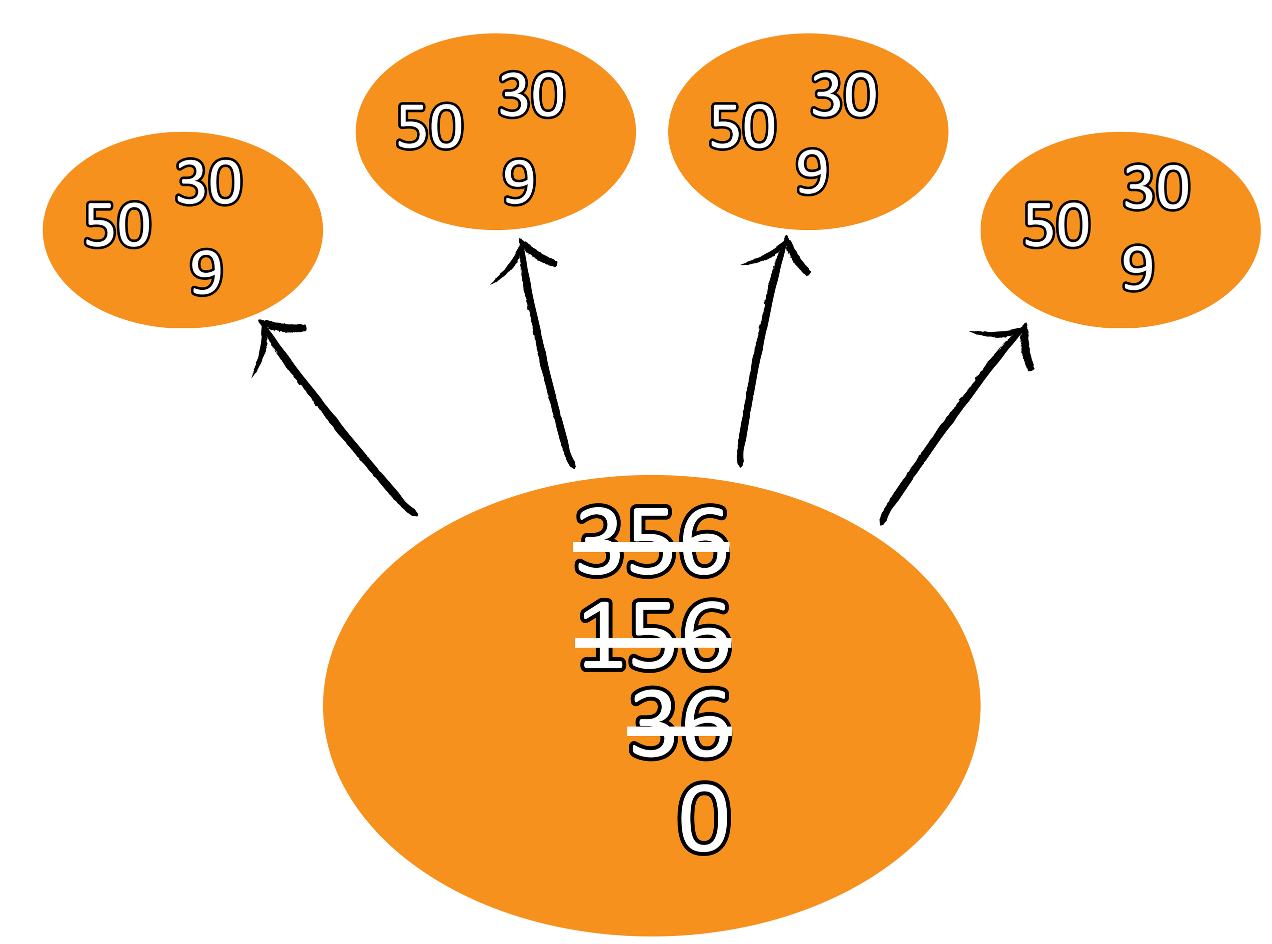
Step 2

	Left	Each
4)	356	
	-200	50
	156	
	-120	30
	36	



Step 3

	Left	Each
4)	356	
	-200	50
	156	
	-120	30
	36	
	-36	+ 9
	0	89



Partial Product Method for Multiplication

Traditional Algorithm

Pros:

- Quickly produces an answer for students with all the skills to carry out the process

Cons:

- The reasoning behind many steps in the process are unclear to students (e.g. multiplying single digits that represent hundreds, regrouping by putting digits above the number and then adding them, etc.)
- When multiplying 2 digits by 2 digits, students find it difficult to keep their regrouping organized and they often don't understand why they start with a "0" when multiplying by the number in the tens place

Go to the next page to see the benefits of the Partial Quotient Method

Partial Product Method

Pros:

- Makes use of expanded form to make the value of the digits clear
- The structure is easier for students to keep organized
- The reasoning behind regrouping is more clear
- The algorithm (symbolic representation) connects very nicely to the area model (pictorial representation). See examples on the next page.
- The multiplication and addition steps are separate and the purpose for each is clear, especially when connected to a concrete or pictorial representation
- Offers many opportunities to teach regrouping, place value, area, the "annexing zeros" strategy, and many other important concepts

Cons:

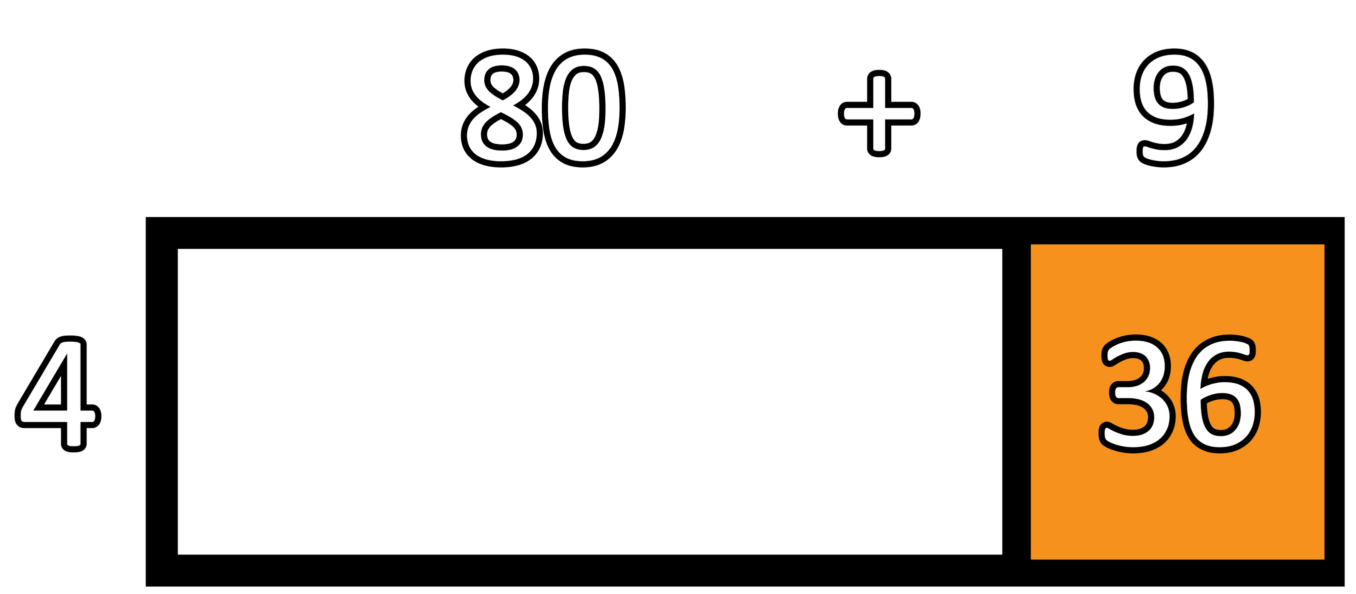
- It's different than the way we learned in school
- It's slightly less efficient than the traditional algorithm for students who are confident with the process

Steps in the Partial Product Method

One digit by two digit example 4x89=

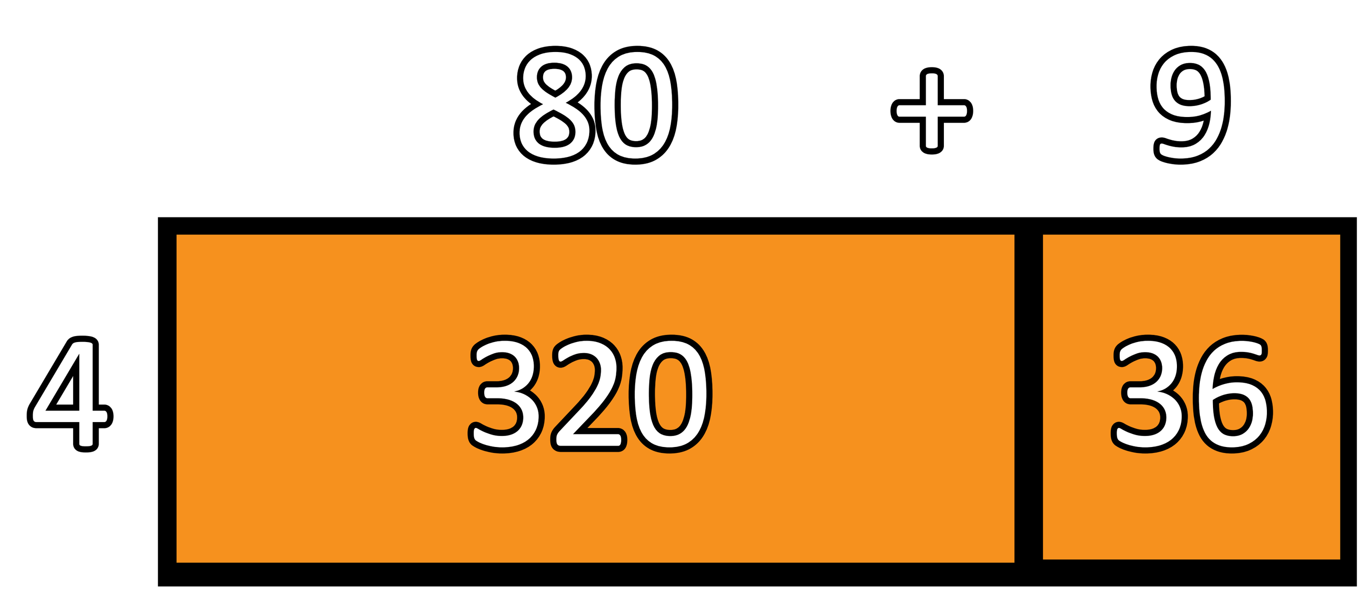
Step 1

```
 80+9
    ↑
  x 4
 ----
   36
```



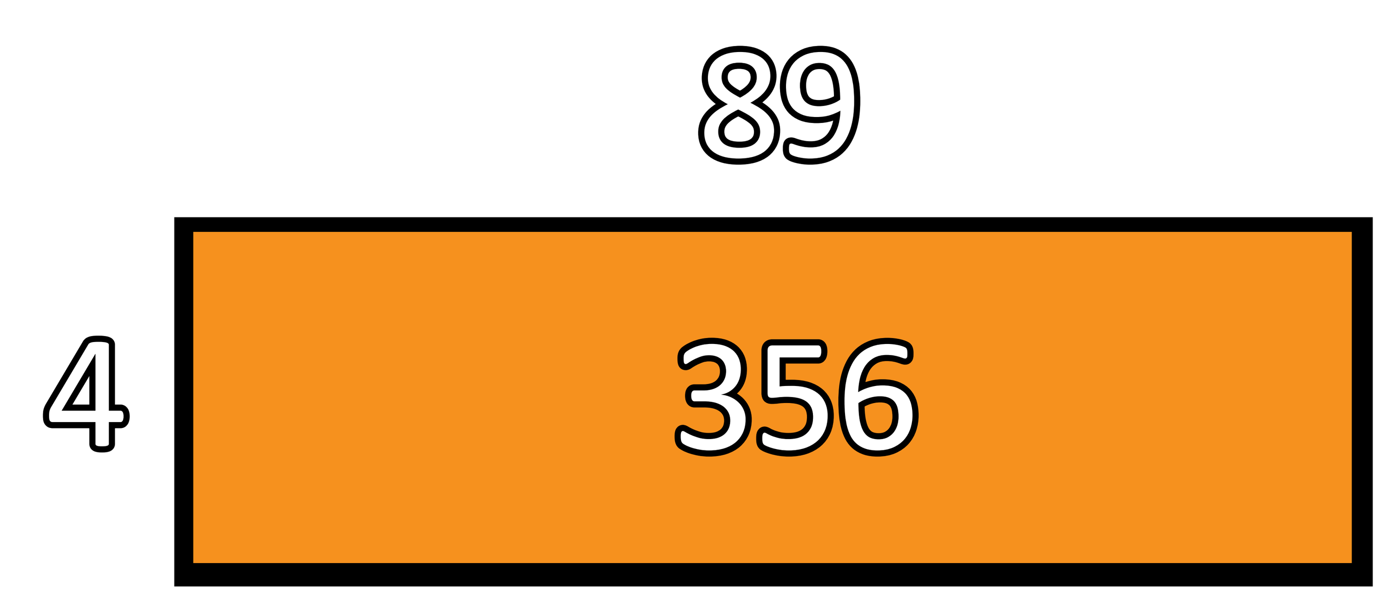
Step 2

```
 80+9
  ↖ ↑
  x 4
 ----
   36
  320
```



Step 3

```
   80+9
    ↖ ↑
    x 4
   ----
     36
  + 320
   ----
    356
```



Steps in the Partial Product Method

Two digit by two digit example (aka “The Bowtie Method”) 43x89=

Step 1

```
   80+9
x  40+3
-------
     27
```

Step 2

```
   80+9
x  40+3
-------
     27
    240
```

Step 3

```
   80+9
x  40+3
-------
     27
    240
    360
```

Step 4

```
   80+9
x  40+3
-------
     27
    240
    360
   3200
```

Step 5

```
   80+9
x  40+3
-------
     27
    240
    360
 +3200
-------
   3827
```