Bar Diagrams

The Logical, Visual Problem-Solving Strategy

Word problems often produce a great amount of anxiety for both students and teachers. To solve a word problem, a variety of knowledge and skills must be utilized: comprehending the problem, applying skills to a new situation, strategizing, computing, and reasoning logically. Using the four-step-problem-solving process adapted from George Polya's plan in conjunction with an appropriate strategy, students can successfully solve problems. The problem-solving steps include:

Understand **Understand the problem.**

- Discuss the given information. Are important facts missing? Is there redundant information?
- Visualize the information. Ask questions to help students form a mental picture of the problem.
- Organize and prioritize the information. Help students restate the facts.
- Connect the information. Encourage students to think about similar problems they have solved before.

Plan **Devise a plan.**

- Invite students to choose the strategy with which they feel most comfortable to solve the problem. More than one strategy may be successfully used to solve a problem.

Solve **Solve, or execute the plan.**

- Encourage students to solve using their plan. If unsuccessful, students can revise and modify the plan, or try a different one.

Check **Check, or reflect on the solution.**

- Have students reflect on their solution by checking it against the problem. Does it make sense?
- Invite students to find an alternate solution.
- Extend the method to other problems. Challenge students to think about what they did to solve more complex problems at a later date.

Commonly used strategies include making a list, looking for patterns, working backward, solving a simpler problem, acting it out, and guessing and checking. However, of the many problem-solving strategies students learn, perhaps the most logical, thorough strategy is *bar diagramming*.

Bar diagrams:

- Help students visualize situations.
- Create concrete pictures from words and abstract situations.
- Demonstrate comprehension of the problem.
- Lead students to a viable solution plan.

Bar diagrams connect problem solving from one grade level to the next. With the continuity of a common method used across different grade levels, students can utilize this strategy with confidence.

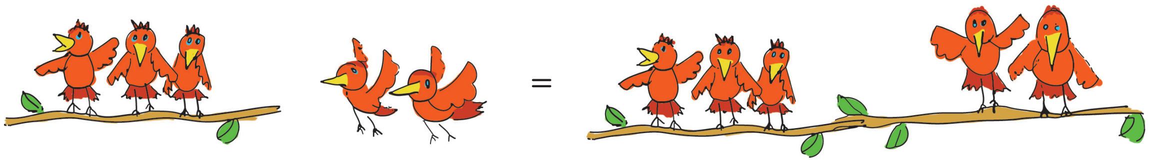
Bar diagrams clearly align with the four-step problem-solving plan.

- Students create a visual representation to demonstrate a clear understanding of the problem.
- The bar diagram requires students to identify the problem's setting and the values associated with the situation.
- The completed bar diagram leads to a definitive solution plan.
- The bar diagram can be used as a visual clue to determine the reasonableness of the solution.

Students can be introduced to bar diagrams as early as Kindergarten. Consider this problem.

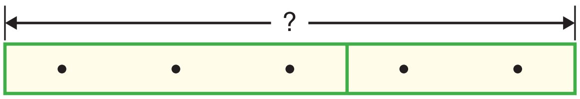
3 birds are on a branch of a tree.
2 birds fly over to the same branch.
How many birds are on the branch now?

Children draw pictures to represent the problem and the solution.

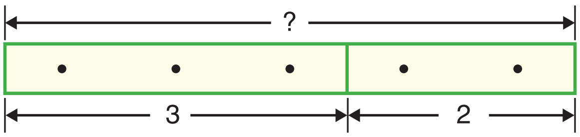


Next, children draw a bar around their visual representation.

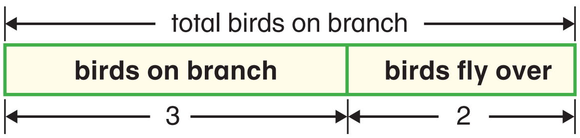
Now, pictures are replaced by dots or squares. The bar showing the total is replaced with an arrow above the "two parts" bar which shows that the whole, or total, is missing.



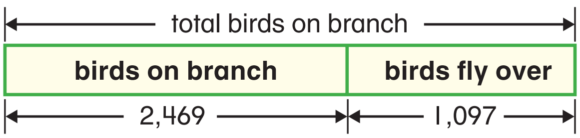
Finally, numbers are recorded beneath the dots.



The dots may be replaced with words that describe the problem's situation.



The words are particularly important in the context of algebraic reasoning. The drawing above can be used to describe the situation of the birds on the branch, regardless of the *number* of birds. For this reason, you may wish to encourage students to complete the words prior to inserting the numbers.



Bar diagrams can be used to represent single-step problems in all four operations as well as multi-step problems. In every problem, students:

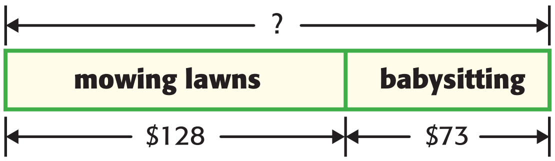
- record the problem's situation in words inside "parts" bars and above the "total" bar
- record the values provided in the problem
- use the bar diagram to identify whether a part or the whole is missing and devise the solution plan

When teaching students how to use bar diagrams, begin with problems containing computation inside students' comfort zone. The practice of bar diagramming may be challenging in itself. Focus students' attention on finding the meaning of the problem and identifying the solution plan revealed by the bar diagram.

The following examples show how the bar diagrams can be drawn to demonstrate comprehension and a solution plan for a variety of problems.

1 One-step addition

Rani earned $128 mowing lawns and $73 babysitting. How much did she earn?



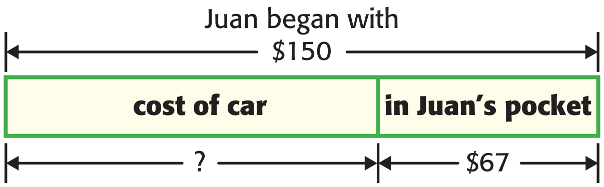
After creating the model drawing, students identify that the whole is missing. They *add* to solve.

$$\begin{array}{r} \$128 \\ +\ 73 \\ \hline \$201 \end{array}$$

Rani earned a total of $201.

2 One-step subtraction

Juan had $67 left after he bought a radio-controlled car. He went to the store with $150. How much did Juan spend on the car?



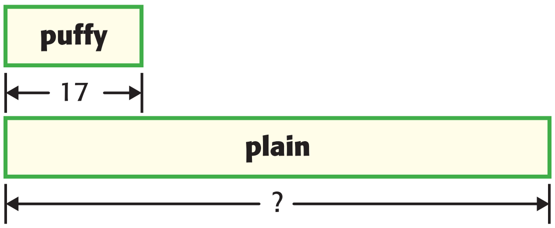
In this problem, the whole is known. That missing part is the car's cost. *Subtract* to find a missing part.

$$\begin{array}{r} \$150 \\ -\ 67 \\ \hline \$\ 83 \end{array}$$

Juan spent $83 on the radio-controlled car.

3 One-step multiplication

Jesse has 17 puffy stickers. She has 4 times as many plain stickers as puffy stickers. How many plain stickers does Jesse have?



This problem is set up as a proportion. There are 4 times as many plain as puffy stickers. *Multiply* by 4 to find the number of plain stickers.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 68 \end{array}$$

Jesse has 68 plain stickers.

4 One-step division

Mackenzie, Sherice, and Darla share the cost of the birthday gift equally. How much does each girl pay?

The whole is known. It is separated into 3 equal parts. Use *division* to find the value of one part.

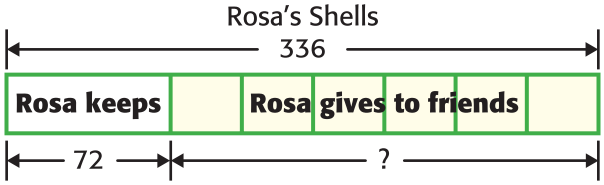
$$\begin{array}{r} 29 \\ 3\overline{)87} \end{array}$$

Each girl pays $29.

Bar diagrams are especially useful in solving multi-step problems. Once drawn, the bar diagram leads the student to the sequence for finding the solution.

5 Whole number operations

Rosa has 336 shells. She keeps 72 of the shells for herself and divides the remaining shells evenly among 6 friends. How many shells does Rosa give each of her friends?



The model drawing reveals the plan:

Step 1: Subtract the shells Rosa keeps from the total.

Step 2: Divide the remaining shells by 6.

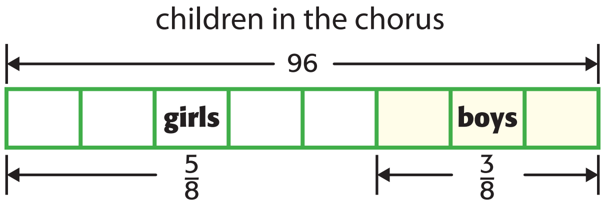
$$\begin{array}{r} 336 \\ -\ 72 \\ \hline 264 \end{array}$$

$$6\overline{)264}\ = 44$$

Rosa gives each friend 44 shells.

6 Fractions

There are 96 students in the chorus. The girls make up $\frac{5}{8}$ of the members. How many boys are in the chorus?



The whole is known. One fractional part is known. **Method 1:** Find the other fractional part. Then find that value of that part relative to the whole.

$1 - \frac{5}{8} = \frac{3}{8}$

$\frac{3}{8} \times 96 = 36$

Method 2: Find the value of one fractional part. Then multiply by the numerator of the fractional part.

$96 \times \frac{1}{8} = 12$

$12 \times 3 = 36$

There are 36 boys in the chorus.

Bar diagramming provides students with a strategy to unpack the structure of a problem and reveal the plan for its solution. The words above and inside the drawing demonstrate students' understanding of the problem's situation. Combined with the values provided in the problem, a solution plan is revealed. After solving the problem, the bar diagram can be used to check the solution by substituting the value into the drawing and using logical reasoning to assess reasonableness.

Bar diagramming is a powerful tool that can be added to any student's problem solving repertoire, regardless of learning style. Its visual and linguistic representation, combined a process utilizing logical reasoning, promote successful problem solving for all students.

Bibliography:

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